НОВЫЙ МЕТОД АНАЛИЗА СОСТОЯНИЯ УПРУГОГО ТЕЛА ОТ МАССОВЫХ СИЛ, ПОРОЖДАЕМЫХ НАНОДИСПЕРСНЫМИ МАГНИТНЫМИ ЖИДКОСТЯМИ

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Реферат. Расчет упругого состояния тела в классической теории упругости традиционно проводят для консервативных массовых сил. Современные технологии позволяют получать упругие материалы с новыми физическими свойствами (например, при пропитке пористых структур нанодисперсными магнитными жидкостями массовые силы формируются воздействием внешнего магнитного поля и могут принимать почти произвольный характер). Метод построения упругих полей, возникающих от неконсервативных массовых сил, теоретически существует, опирается на интегральные представления через ядра Гринга и поэтому не эффективен. В статье представлен и протестирован новый эффективный для практических целей метод построения упругих полей от равномерно-непрерывных сил.

Ключевые слова: нанодисперсные магнитные жидкости, массовые силы, гильбертово пространство, напряженно-деформированное состояние, непотенциальные силы.

A NEW METHOD FOR ANALYZING THE EFFECT OF BODY FORCES INDUCED BY NANODISPERSED MAGNETIC FLUIDS ON STATES OF ELASTIC SOLIDS

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Abstract. The calculation of the elastic state of the body of the classical theory of elasticity is traditionally performed for conservative mass forces. Modern technologies allow to obtain elastic materials with advanced physical properties (e.g., impregnation of the traditional patterns by nanodispersed magnetic fluids) what mass forces are generated by the external magnetic field and can take almost arbitrary character. Method of evaluation of elastic fields, caused from non-conservative mass forces, theoretically exists, relies on the integral nebulosity with kernels of Grin and therefore not effective. In the article, a new efficient for practical purposes method of construction of field due to the non-conservative forces of continuous character is suggested, tested and proved out.

Keywords: nanodispersed magnetic fluids, body forces, Hilbert space, stress-strain states, non-potential force.

Introduction. The observed acceleration of scientific-technical progress has already led to the creation of new materials (composites), effectively used in aerospace industries. These trends in the near future will certainly be gaining ground in other branches, in particular for the improvement of agricultural machinery. A promising technology is the impregnation of the porous framework of nanodispersed fluids, after curing, finished product will have significant functional advantages over their conventional counterparts, but will get also side effects, which will become a necessity for designers, while effective techniques for this do not exist yet. For example, when using nanodispersed magnetic fluids under the influence of external electromagnetic fields will arise the body force in the body construction, the neglect of which is unacceptable from the standpoint of ensuring strength. Such forces obviously will not have a potential nature. Existing effective methods of evaluation of the stress-strain state are guided by the conservative mass forces (gravitational, inertial). Development of a method to analyze the impact of the power factors of the nonconservative nature of the task is current, relevant.

1. Description of the object and method of research

Analysis of linear isotropic elastostatic fields commonly factors in the following basic medium equations: Cauchy equations, Hooke’s law, and the equilibrium equation

$$\sigma_{ij,j} + \rho K_i = 0,$$

(1)

where $\sigma_{ij}$ is the stress tensor component, $\rho$ is the density of the material, and $K_i$ is the body force. The next step is decomposition with the solution of the boundary problem presented as the sum of two stress-strain states (SSS), one of which is determined by body forces and the other by the same basic equations as above, but with $K_i = 0$. The correction from the first solution is taken in boundary conditions. In classical scenarios body forces are assumed to be conservative, as providing an example of a non-conservative force in pure mechanics is a challenge. This makes finding a field corresponding to body forces quite easy [1].

New materials produced by nanotechnology, in particular those produced by impregnation of traditional structures with nanodispersed magnetic fluids, experience the occurrence of random body forces induced by external electromagnetic fields. In theory, there is an extremely inconvenient method for analyzing states (based on random forces) using Green tensor [2], which involves the calculation of infeasible integrals and all it entails.
A general method that would make it possible to efficiently tackle the difficulties commonly arising from the existence of continuous non-conservative body forces would therefore be welcome. Such a method would be contingent on the solution of the following interrelated problems: justification of the inverse method for building SSS based on continuous body forces and solution of related specific problems. The authors have devised such a method, with the summary provided below:

Since a displacement field in a continuous solid cannot be discontinuous, every component of the displacement vector may be represented as a series based on the Weierstrass fundamental polynomial system. The monomial \( w = x^a y^b z^c \) may be placed in any position of the displacement vector \( u(x, y, z) \) creating a certain acceptable elastic state. E.g., in the scenario \( u = (w, 0, 0) \) we use the sequence "Cauchy formula – generalized Hooke's law" to determine the relevant deformation and stress tensors

\[
\hat{\mathbf{e}} = \frac{w}{2} \begin{pmatrix} 2\alpha x^{-1} & \beta y^{-1} & \gamma z^{-1} \\ \beta y^{-1} & 0 & 0 \\ \gamma z^{-1} & 0 & 0 \end{pmatrix}, \quad \hat{\mathbf{\sigma}} = w \begin{pmatrix} (\lambda + 2\mu)\alpha x^{-1} & \mu \beta y^{-1} & \mu \gamma z^{-1} \\ \mu \beta y^{-1} & \lambda \alpha x^{-1} & 0 \\ \mu \gamma z^{-1} & 0 & \lambda \alpha x^{-1} \end{pmatrix},
\]

where \( \lambda \) and \( \mu \) are Lamé constants. The equilibrium equation (1) allows for formulating the equation for calculating the body force

\[
\mathbf{K} = -\frac{w}{\rho} \begin{pmatrix} (\lambda + 2\mu)\alpha (\alpha - 1)x^{-2} + \mu \beta (\beta - 1)y^{-2} + \mu \gamma (\gamma - 1)z^{-2} \\ (\lambda + \mu)\alpha \beta x^{-1}y^{-1} \\ (\lambda + \mu)\alpha \gamma x^{-1}z^{-1} \end{pmatrix},
\]

ensuring the closure of relations.

By varying \( a, \beta, \gamma \) we can devise a set of such states. Various alternatives within \( a + \beta + \gamma \leq n \) produce a multitude of body force vectors, whose components are represented by homogeneous polynomials up to and including the degree of \( n-2 \). Does this multitude contain a sufficient basis set for us to present any vector of continuous body forces as a Fourier series using the elements of this basis?

By generating all the alternatives within the cluster \( n = a + \beta + \gamma \) and testing their linear independence by means of numerical experiments for \( n = 2 \ldots 12 \), the authors established the dependence of the number \( 3m \) of linearly independent alternatives of the vector \( \mathbf{K} \) on the cluster number \( n \) (table 1):

<table>
<thead>
<tr>
<th>( n )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3m )</td>
<td>3</td>
<td>9</td>
<td>18</td>
<td>30</td>
<td>45</td>
<td>63</td>
<td>84</td>
<td>108</td>
<td>135</td>
<td>165</td>
<td>198</td>
</tr>
</tbody>
</table>

The cluster \( n \) contains \( (n-1)n/2 \) linearly independent polynomials, which coincides with the number of linearly independent monomials of the degree \( n-2 \). It follows that the basis set of polynomials approximating a random polynomial of the degree \( n-2 \) is complete. Thus a numerical experiment provided evidence for the availability of a basis set of vectors generated by various polynomials up to and including the degree of 10. The number of linearly independent vectors generated as per this pattern (up to the cluster \( n \)) and approximated to the degree of polynomials \( n-2 \) equals \( \frac{3}{2} \sum_{i=1}^{n-1} i(i+1) \).

The basis set can be further orthogonalized within the area \( V \) with respect to the dot product
\[ \left( K^{(1)}, K^{(2)} \right) = \int_V K^{(1)} \cdot K^{(2)} \, dV \]

Any continuous vector of body forces may be represented as a Fourier series of elements of \( K(k) \) of the orthonormal basis set:

\[ K = \sum_{k=1}^{\infty} c_k K^{(k)}, \quad c_k = \left( K, K^{(k)} \right). \]

We further assess the approximation error using Bessel's inequality:

\[ \delta = \sqrt{(K, K)} - \sqrt{\sum_{k=1}^{N} c_k^2}. \]

The value \( \delta \) is used as reference for selecting the length \( N \) of the retained interval of the basis set of the Hilbert space of body forces.

Note. Provided that body forces and density are represented as polynomials, the above algorithm results in precise SSS modeling (with no representation errors).

Each basis vector \( K^{(k)} \) used in the model corresponds to a displacement vector \( u^{(k)} \) and deformation and stress tensors \( \hat{\varepsilon}^{(k)} \) and \( \hat{\sigma}^{(k)} \), which collectively form an internal state

\[ \xi^{(k)} = \{ u^{(k)}, \hat{\varepsilon}^{(k)}, \hat{\sigma}^{(k)} \}. \]

Any linear combination of vectors \( K^{(k)} \) corresponds to a similar combination of states \( \xi^{(k)} \). I.e., by orthogonalizing a body force and expanding it into a series we can efficiently build a linear combination determining the SSS \( \xi \) conditioned by the body force \( K \).

### 2. Description and analysis of the results

As an example, we shall analyze the expansion of a non-potential force

\[ \rho K = \left\{ 0, 0, -2\sqrt{x^2 + y^2} \right\} \]

into a series determined by a basis set interval containing 495 elements. We omit the basis set interval, the result of orthogonalization of the cube \(-1 \leq x, y, z \leq 1\), the result of expansion into a Fourier series, and relevant SSS \( \xi \) due to the awkwardness of the required mathematical operations. We shall confine our case to comparing the curves of the given force \( \rho K_3 \) and its expansion plotted on the main diagonal \( x=y=z \) joining the corners of the cube (Fig. 1).

![Fig. 1 – Comparing given and estimated values of a body force: dotted line for approximation and solid line for given value](image)

As the curves show, the difference between the two values being compared is marginal. The example analysed testifies to the practical efficiency of the new method for SSS assessment based on non-conservative body forces.
Список литературы

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